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Robust Localization Methods for Passivity Enforcement of Linear Macromodels

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***Abstract*—In this paper we solve a non-smooth convex formulation for passivity enforcement of linear macromodels using robust localization based algorithms such as the ellipsoid and the cutting plane methods. Differently from existing perturbation based techniques, we solve the formulation based on the direct $\mathcal{H}_\infty$ norm minimization through perturbation of state-space model parameters. We provide a systematic way of defining an initial set which is guaranteed to contain the global optimum. We also provide a lower bound on the global minimum, that grows tighter at each iteration and hence guarantees $\delta - optimality$ of the computed solution. We demonstrate the robustness of our implementation by generating accurate passive models for challenging examples for which existing algorithms either failed or exhibited extremely slow convergence.**

I. Introduction

The generation of linear macromodels for electrical interconnects is a standard practice in signal and power integrity analysis. Starting from tabulated frequency samples of the scattering matrix obtained from either measurements or full-wave analysis, a macromodel is generated, which can in turn be synthesized into an equivalent circuit or an equation-based state-space macromodel for fast system-level simulations and design optimization. Passivity of these models is a fundamental requirement that guarantees numerically stable system-level simulations.

Generating a *guaranteed* passive model from available frequency response data is a challenging task, mainly because the problem is nonlinear and non-convex. Convex relaxations to this problem are proposed in [1]–[4]. These algorithms rely on enforcing passivity by defining constraints for the positive real lemma or the bounded real lemma. Although these constraints can be certifiably enforced, it is normally a costly operation.

Researchers have been working on an iterative perturbation framework such as [5], [6]. In these techniques a stable but not necessarily passive model is first identified using rational fitting techniques such as vector fitting [7]. This model is then checked for passivity violations by examining the corresponding Hamiltonian matrix. Finally, some parameters of the initially identified non-passive model are perturbed to correct for passivity violations. A comprehensive comparison of the main techniques is available in [8]. These techniques are computationally efficient however their main drawback is that their formulation does not guarantee convergence of the algorithm or optimality of the solution.

In this paper we use robust localization based algorithms, such as the ellipsoid algorithm [9] and the cutting plane method [10], to solve the convex continuous but non-smooth passivity enforcement formulation presented in [11]. The algorithms presented in [11] exhibit high sensitivity to given problem parameters and need tuning of the algorithm coefficients for individual cases. Compared to [11], the algorithms implemented in this paper are robust in the sense that they are less sensitive to the given problem parameters and converge with in acceptable number of iterations even for the challenging cases where the subgradient techniques in [11] converge too slowly. In this paper, we also provide a scheme to determine the initial set which is guaranteed to contain the global solution of the problem. Furthermore when using the cutting plane method, we provide a lower bound on the global minimum which becomes tighter as the number of iterations increase. This lower bound is used to define stopping criteria for the algorithm. It may also be used to infer the quality of perturbed models with fixed poles. The algorithms implemented in this paper are guaranteed to converge to the global minimum and with the cutting plane method we can additionally guarantee $\delta - suboptimality$ of the solution. We use these algorithms to solve two relatively small but challenging problems that arise in interconnect modeling. For these two examples, existing perturbation based approaches such as [6] failed to converge while the algorithms presented in [11] demonstrated extremely slow convergence.

II. Background: Problem Description

We use the same problem formulation as proposed in [11] and is summarized in this section. Consider a nominal state-space macromodel characterized by its $n_y \times n_u$ transfer matrix

$$H(0,s) = C(sI-A)^{-1}B + D \qquad (1)$$

The macromodel $H(0,s)$ is generally available through stable model identification of given frequency response samples $\{(\omega_k, S_k), k = 1,...,K\}$ of the scattering matrix for some linear device such as a filter or an electrical interconnect. Rational approximation techniques such as *Vector Fitting* [7] can be applied to these samples to find the state-space macromodel (1). System (1) is assumed to be asymptotically stable and state-space realization is assumed to be minimal. A stable system (1) is passive if and only if

$$||H(0)||_{\mathcal{H}_\infty} = \sup_{\omega\in\mathbb{R}} \sigma_1(H(0, j\omega)) \leq 1 \qquad (2)$$

here σ_1 denotes the largest singular value. In the cases where (2) does not hold, the state space matrices are perturbed such that the resulting perturbed system is passive. A common choice is to perturb only the state-space C matrix which usually stores the residues for the partial fraction expansion of $H(0,s)$. Matrix D, which corresponds to the high frequency response $(s \to \infty)$, is assumed to have $||D||_2 = \sigma_1(D) \leq 1$, which is necessary for passivity. The perturbed system is defined as

$$H(C_P,s) = (C+C_P)(sI-A)^{-1}B+D \tag{3}$$

where the perturbation matrix C_P is unknown. Supposing that the original system $H(0,s)$ is *not* passive, the goal here is to find the minimal perturbation such that the perturbed system $H(C_P,s)$ is passive. The problem can be formulated as

$$\underset{C_P}{\text{minimize}} \ ||C_P||_F \quad \text{s.t.} \quad ||H(C_P)||_{\mathcal{H}_\infty} \leq 1, \tag{4}$$

where the minimal perturbation condition is expressed, without the loss of generality, in terms of the Frobenius norm. Defining the cost function as euclidean distance of the perturbation point from the origin offers additional intuitive value in defining initial hypersphere containing the global minimizer of (4) (explained in Section III-A). Other weighted norms can be used with trivial extension. Using vectorized variable $x = vec(C_P) \in \mathbb{R}^n$ and rewriting (4)

$$\underset{x}{\text{minimize}} \ f(x) \quad \text{s.t.} \quad h(x) \leq 1, \tag{5}$$

here $f(x) = ||x||_2 = ||C_P||_F$ and $h(x) = ||H(C_P)||_{\mathcal{H}_\infty}$. Problem (5) admits a global minimum if both f and h are convex and the problem is feasible. For the problem in hand, both f and h are norms and, by virtue of triangular inequality, are convex. Furthermore this problem is feasible since one can always find at least one feasible point, namely $x = -x_c(x_c = vec(C))$. This is because $x = -x_c \implies H(C_P,s) = H(-C,s) = D$ which is assumed to be passive. As described in [11], $h(x)$ is continuous but non-smooth. The subdifferentials of $h(x)$ are derived in [11] and are not repeated here.

III. Solution Using Localization Methods

In this paper we propose using robust localization based algorithms such as the ellipsoid and the cutting plane methods to solve (5). Localization based methods are the ones where the set containing the global optimum becomes smaller at each step. For such algorithms, we need to define an initial set that is guaranteed to contain the global optimum.

A. Initial Set Guaranteed to Contain Global Optimum

One of the main challenges in using localization methods is to come up an initial set that is guaranteed to contain global optimum. In this section we define an initial set in the form of a hypersphere that is guaranteed to contain the global minimum for our problem.

Lemma 3.1: Let $f(x) = ||x||_2$ be the cost function, and let the feasible set be defined by $h(x) \leq 1$. Suppose that we are given *any* feasible point x_f (i.e. $h(x_f) \leq 1$). Then the hypersphere centered at origin with radius equal to the euclidean distance of x_f from the origin is guaranteed to contain the global optimum.

Proof: Let x_f be the given feasible point. We define the initial hypersphere with radius $R = ||x_f||_2$. Now suppose that the global optimal solution x^* lies outside the hypersphere, which by definition requires $||x^*||_2 > ||x_f||_2$. This means $f(x^*) > f(x_f)$ which leads to a contradiction because the condition of optimality requires $f(x^*) \leq f(x_f)$. ■

As described in Section II, one of such feasible points is $x = -x_c$. We can define radius of the initial hypersphere $R = ||x_c||_2$. In fact we can use this feasible point to find an even smaller radius using simple line search as

$$\beta^* = \underset{\beta \in [-1,0]}{\text{argmin}} \ f(\beta x_c) \quad \text{s.t.} \quad h(\beta x_c) \leq 1 \tag{6}$$

$$R = ||\beta^* x_c||_2 \tag{7}$$

Note that the methods described in this paper can be used to improve the accuracy of an existing perturbed passive system. For such systems, the solution which is feasible but inaccurate can be used to define the initial hypersphere and the algorithms described in this paper can be used to improve accuracy by providing global optimum. Additionally we provide lower bounds on the global minimum which can be used to assess quality of the available solutions.

B. Solution Using The Ellipsoid Algorithm

The ellipsoid algorithm [9] belongs to the class of *localization* methods. We provide an intuitive description of the ellipsoid algorithm to solve (5). We start by defining an initial ellipsoid, ε_0, which is guaranteed to contain the global optimal x^*. We define ε_0 to be the hypershpere, $P_0 = R^2 I_n$, where the radius R computed by (7). At each iteration the gradient

$$g_x \in \begin{cases} \partial f(x) & \text{if} \quad h(x) \leq 1 \\ \partial h(x) & \text{if} \quad h(x) > 1, \end{cases} \tag{8}$$

defines a hyperplane that divides the whole space in two half spaces, with the global optimal x^* lying in $\mathcal{H}_- = \{z : g_x^T(z-x) < 0\}$. We use this information to update the ellipsoid such that the updated ellipsoid contains x^* and vol $\varepsilon_{k+1} <$ vol ε_k (refer to [9] for further details). The ellipsoid method is efficient and robust but it may exhibit slower convergence because the volume reduction factor depends on n (size of x) i.e., vol $\varepsilon_{k+1} < e^{-\frac{1}{2n}}$vol ε_k. The update cost for ellipsoid algorithm is $O(n^2)$ which is very cheap computationally.

C. Solution Using The Cutting Plane Method

The cutting plane method [10] also belongs to the class of *localization* methods. We define the initial set (a polyhedron) containing the global optimal x^* to be the hypercube, $\mathcal{P}_0$, centered around the origin with *side length* $= 2R$ (R is computed using (7)). We shall refer to the size of this hypercube as R. At each step we add a new constraint to the current polyhedron $\mathcal{P}_k$ such that the updated polyhedron $\mathcal{P}_{k+1}$ is smaller and contains the global minimum. Algorithm 1 describes the cutting plane method conceptually. Note that we want x_{k+1} to be near the center of $\mathcal{P}_{k+1}$ so that the next polyhedron is as small as possible. There are various ways to compute x_{k+1} each having cons and pros. We choose x_{k+1} to be the analytic center of the inequalities, defining $\mathcal{P}_{k+1} = \{z | a_i^T z \leq b_i, \ i = 1...q\}$, which gives a well rounded performance both in terms of computational cost and convergence. We compute the analytic center by solving (9) using infeasible start Newton method

$$x_{k+1} = \underset{x}{\text{argmin}} - \sum_i log(b_i - a_i^T x) \tag{9}$$

Algorithm 1 Cutting Plane Method

Input: f, h and $\mathcal{P}_0$ such that $x^* \in \mathcal{P}_0$, where $\mathcal{P}_0$ is a hypercube of size R (computed using (7)) centered around origin; initialize the lower bound $L_0 = 0$ and $f_{best} = f(x_{feasible})$; accuracy $\delta > 0$,set $k = 0$

1: If $h(x_k) \leq 1$, let $g_k \in \partial f(x_k)$, else $g_k \in \partial h(x_k)$
2: Query the cutting plane oracle at x_k
3: If $h(x_k) \leq 1$ and $|f_{best} - L_{best}| < \delta$, then return x_k and quit
4: Update $\mathcal{P}$: add a new cutting plane $a_{k+1}^T z \leq b_{k+1}$, $\mathcal{P}_{k+1} := \mathcal{P}_k \bigcap \{z | a_{k+1}^T z \leq b_{k+1}\}$ here $a_{k+1} = g_k$ and if feasible $b_{k+1} = g^T x_k$ else $b_{k+1} = g^T x_k - h(x_k) + 1$
5: Update $x_{k+1} \in \mathcal{P}_{k+1}$, L_k
6: Update $L_{best} = \max_k L_k$, $f_{best} = \min_k f(x_{k,feasible})$
7: Let $k \leftarrow k+1$ and goto 1.

Piecewise Linear Lower Bound on the Global Minimum: Suppose the function f and its gradient ∂f has been evaluated at $x_0, x_1, \ldots, x_k$. Since f is convex, one can compute the linear under estimators of f as follows

$$\begin{aligned} f(z) &\geq f(x_i) + g_i^T (z - x_i), \quad \forall z, i = 1, \ldots, k \\ \implies f(z) &\geq \hat{f}(z) = \max_{i=1,\ldots,k} \left(f(x_i) + g_i^T (z - x_i) \right) \end{aligned} \tag{10}$$

Here $\hat{f}(z)$ is a convex piecewise linear global under estimator of f. Similarly one can also find piecewise linear approximations, $\hat{h}(x) \leq 1$, of the constraint $h(x) \leq 1$. This helps formulating a piecewise linear relaxed problem (11) which can be solved using standard linear programming solvers.

$$x_{LB}^* = \text{argmin } \hat{f}(x) \quad s.t. \; \hat{h}(x) \leq 1 \tag{11}$$

$L_k = ||x_{LB}^*||_2$ defines a lower bound on the global minimum of (5). This lower bound gets tighter with increasing iteration and is used to define the stopping criteria. The cutting plane method is *guaranteed* to provide the global minimum with the given accuracy. Note that the linear program (11) is rarely solved to compute the lower bound. The lower bound can be easily computed as a by product when solving for the analytic center of the inequalities.

IV. Results

In this section we provide two simple but challenging examples that arise in enforcing passivity. In example 1 and 2, the given stable but non passive models exhibit large passivity violations and passivity violations over a wide bandwidth, respectively. We used two alternative approaches [6] and [11] for comparison, both of which failed to provide satisfactory solutions. The scheme in [6] diverged for both cases. The algorithm based on subgradient methods, presented in [11] exhibited extremely slow convergence for both of the cases. Additionally, the algorithm presented in [11] is not robust and highly sensitive to problem parameters and the algorithm breaks if for example the largest singular value of the transfer function is found to be asymptotically at infinity. It takes fraction of a second per iteration for both algorithms to solve on a desktop machine with Intel Xeon processor with 2.4GHz clock without exploiting parallel programming.

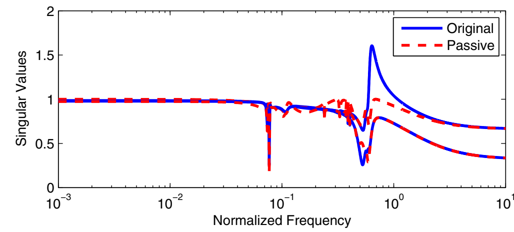

Fig. 1. Example 1: Singular values of the original non-passive (solid blue) and perturbed passive (dashed red) models plotted against normalized frequency

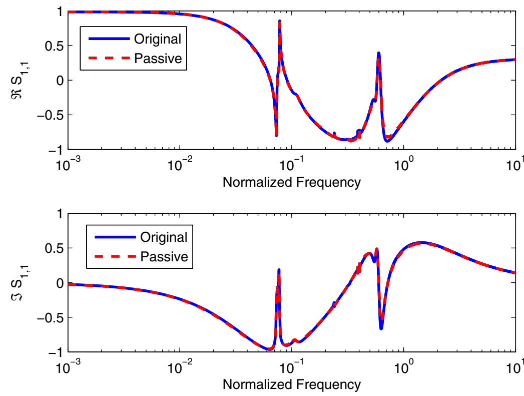

Fig. 2. Example 1: Comparison between original non-passive and perturbed passive scattering response $S_{1,1}$

A. Example 1: Large Passivity Violation

We consider a practical system for which the initial stable but non passive model has large passivity violations as shown by the blue solid curves in Figure 1. It is a testcase with $2-ports$, $36-poles$, $C \in R^{2\times 72}$ and the unknown vector $x = vec(C_P) \in R^{144}$. Figure 2 compares the scattering response $S_{1,1}$ of the original non-passive model with the perturbed passive model. To improve accuracy in the frequency response, we use the Cholesky factor of the controllability Gramian to define weights on the cost function as described in [5]. For this testcase the radius computed using (7) is $R \approx 1.0$. First we solve the problem using the cutting plane method. Even though we would like the initial hypercube to be as small as possible, in our experiments we vary the size of initial hypercube to demonstrate that the convergence of the cutting plane method is not too sensitive to the initial size. We define the initial hypercubes with sizes $R \in \{10.0, 1.0, 0.06\}$. All of these hypercubes are guaranteed to contain the global optimal. Figure 3 shows the convergence of the cost function for different initial hypercubes. We note that increasing the size from 1 to 10 did not have a significant impact on the convergence and the algorithm converged in about 400 iterations. We compute the lower bound on the cost function, by solving (11), which is shown by the black curve in Figure 3. Using this lower bound, we can guarantee that the computed solution is $\delta - suboptimal$.

Next we solve the problem using the ellipsoid method with initial hyperspeheres having $R \in \{0.15, 0.1, 0.06\}$. From Figure 4, we note that the ellipsoid method exhibits slower convergence than the cutting plane method. The ellipsoid

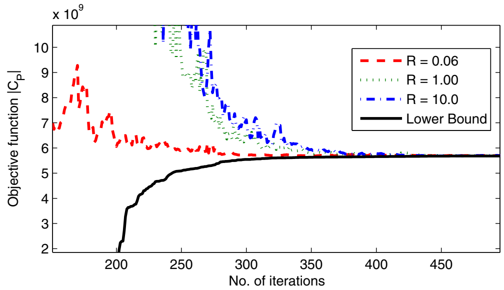

Fig. 3. Example 1: Cutting plane method using analytic centering. The cost function plotted against the iteration number for different initial hypercubes

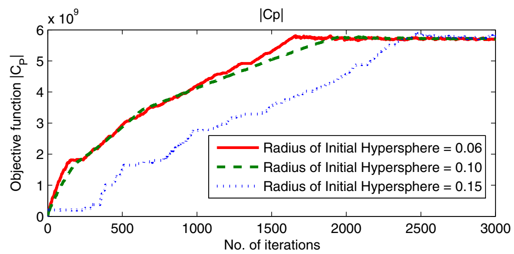

Fig. 4. Example 1: Ellipsoid method. The cost function plotted against the iteration number for different initial hyperspheres

method is cheaper but more sensitive to size of the initial hypersphere as Figure 4 shows. E.g. increasing the initial radius from 0.06 to 0.15 costs about 1000 iterations whereas a similar change in the initial size did not have a noticeable impact on the convergence of the cutting plane method.

B. Example 2: Wide Bandwidth Passivity Violation

We consider a practical system for which the initial stable but non passive model has passivity violations over a wide bandwidth as shown by the blue solid curves in Figure 5. Such a behavior poses serious challenges to existing perturbation based approaches. It is a testcase with $2-ports$, $34-poles$, $C \in R^{2\times 68}$ and the unknown vector $x = vec(C_P) \in R^{136}$. The size of initial hypercube computed by (7) is $R \approx 2.4$. Figures 6 shows the convergence of the cutting plane method to global minimum for different initial hypercubes.It also plots the piecewise linear lower bound on the global minimum. Performing similar tests with the ellipsoid algorithm, we arrive at the conclusion that the convergence of the cutting plane method is less sensitive to the initial set than that of the ellipsoid algorithm. Both methods are robust and converge to the global minimum.

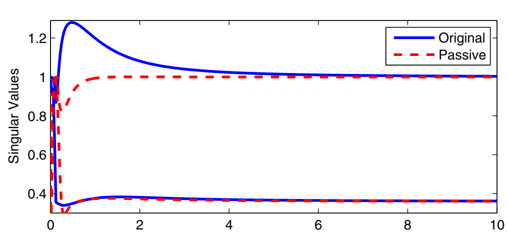

Fig. 5. Example 2: Singular values of the original non-passive (solid blue) and perturbed passive (dashed red) models plotted against normalized frequency

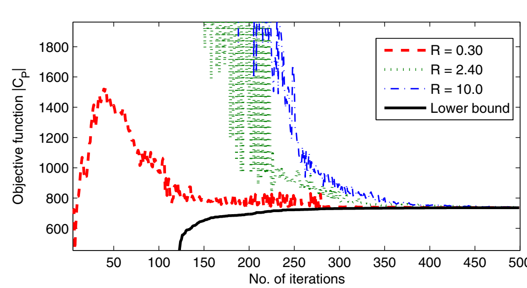

Fig. 6. Example 2: Cutting plane method using analytic centering. The cost function plotted against the iteration number for different initial hypercubes

V. Conclusion

In this paper we employ the robust localization based methods to solve the formulation of passivity enforcement presented in [11]. We provide a technique to compute a smaller initial set which is guaranteed to enclose the global optimal. We also provide lower bound on the global minimum which becomes tighter at each iteration. For the cutting plane method, we guarantee that the solution can be found up to any prescribed accuracy within a finite number of iterations. Furthermore the lower bound can be used to assess quality of all possible solutions that could be found with fixed poles. These schemes are robust and converge for challenging cases where existing approaches fail to find a solution.

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